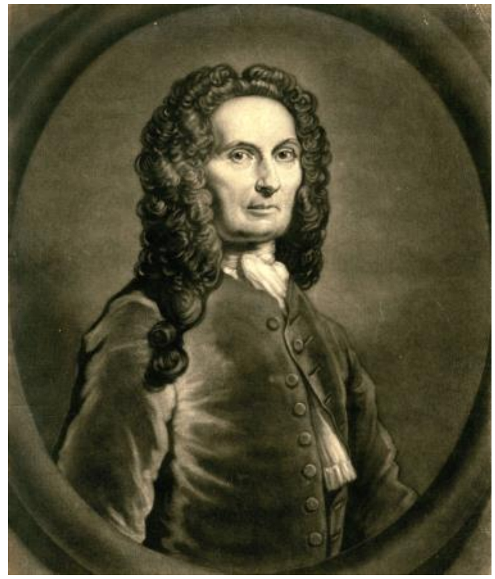
Warm-up question, Lecture 2

Tuesday, January 12, 2021 2:39 PM



Abraham De Moivre

Suppose that the function $f = u + \boldsymbol{i}v\colon \boldsymbol{R}^2 \to \boldsymbol{R}^2$ has all the first partial derivatives for all points $z \in B(z_0, r)$.
What can you say about f?

1. f is continuous in $B(z_0, r)$, but not always differentiable.
2. f is differentiable in $B(z_0, r)$.
3. The functions u and v are differentiable in $B(z_0, r)$.
4. f can be discontinuous at some points of $B(z_0, r)$.

$$z = x + iy$$

$$f(z) = \begin{cases} \dfrac{xy}{x^2+y^2} + i0, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y}\Big|_0 = 0$$

Not continuous at 0: $z = \varepsilon(1+i)$ $f(z) = \dfrac{\varepsilon^2}{2\varepsilon^2} = \dfrac{1}{2} \not\to 0$

Warm-up question. Lecture 3

Thursday, January 14, 2021 9:22 PM

What is $\lim_{z\to -\frac{1}{2}} \frac{\{z^2-\frac{1}{4}\}}{\{2z+1\}}$?

1) ∞
2) 0
3) $-\frac{1}{2}$
4) The limit does not exist.

$$\frac{z^2-\frac{1}{4}}{2z+1} = \frac{(z-\frac{1}{2})\cancel{(z+\frac{1}{2})}}{2\cancel{(z+\frac{1}{2})}}$$

#3

Warm up question. Lecture 4.

Tuesday, January 19, 2021 11:10 PM

What is the $\limsup_{n\to\infty}((-1)^n + \frac{\sin n}{n})$?

1) 1
2) -1
3) Does not exist.
4) What is limsup?

Warm up question. Lecture 5

Friday, January 22, 2021 1:32 PM

For which values of α the series $\sum \frac{(-1)^n z^n}{n^\alpha}$ converges uniformly in the closed unit disk?

1. $\alpha > 0$
2. $\alpha > 1$
3. All values of α.
4. It never converges uniformly in the closed unit disk.

$\alpha \leq 1$ $\underbrace{z = -1}$ $\sum \frac{(-1)^n(-1)^n}{n^\alpha} = \sum \frac{1}{n^\alpha}$ - diverges

$\alpha > 0$ Converges at all z, $|z| \leq 1$, $z \neq -1$. Bonus, +1 pt.

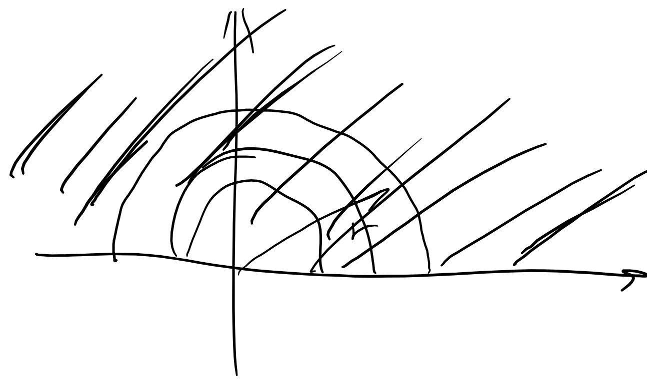
$\alpha > 1$ $\underbrace{\frac{|(-1)^n z^n|}{n^\alpha} \leq \frac{1}{n^\alpha}}$ $(\underline{\underline{|z| \leq 1}})$ By M-test, since $\sum \frac{1}{n^\alpha} < \infty \Rightarrow \sum \frac{(-1)^n z^n}{n^\alpha}$ converges uniformly.

Warm up question. Lecture 6.

Thursday, January 28, 2021 7:07 AM

What is the interior of the closure of the set $\{z \in \boldsymbol{C} : |z| \in \boldsymbol{Q},\ \operatorname{Arg}(z) > 0\}$?

1. ~~The complex plane~~ $\boldsymbol{C}$.
2. A half-plane $\{\Im z > 0\}$.
3. $\emptyset$.
4. I don't know.



$r \in \mathbb{Q}$

Warm up question. Lecture 7

Friday, January 29, 2021 10:23 AM

What is the differential of the function $f(z) = z^2$?

1) $df = 2xdx + \boldsymbol{i}2ydy$
2) $df = (2x + \boldsymbol{i}2y)dx + (2x + \boldsymbol{i}2y)dy$
3) $df = (2x + \boldsymbol{i}2y)dx + (-2y + \boldsymbol{i}2x)dy$
4) What is "the differential"?

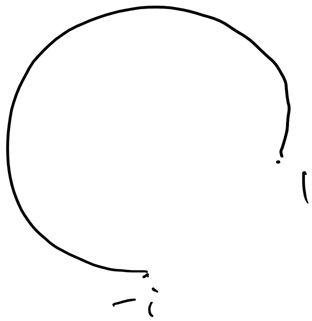
$$dz^2 = 2z\,dz = 2(x+iy)(dx+idy)$$

Warm up question. Lecture 8.

Wednesday, February 3, 2021 9:54 AM

Compute $\oint_\gamma xdx - ydy$, where γ is a the longer arc of the unit circle joining $-\boldsymbol{i}$ with 1.

1) 0
2) -1
3) 1
4) -1/2



$$x\,dx - y\,dy = d\left(\frac{x^2-y^2}{2}\right)$$

$$\int x\,dx - y\,dy = \left.\frac{x^2-y^2}{2}\right|_{-i=(0,-1)}^{1=(1,0)} =$$

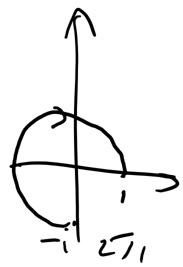
$$\frac{1}{2} - \left(-\frac{1}{2}\right) = 1.$$

Warm up question. Lecture 9.

Saturday, February 6, 2021 10:32 AM

Compute $\oint_\gamma \bar{z}^2 dz$, where γ is a the longer arc of the unit circle joining $-\boldsymbol{i}$ with 1.

1) 0.
2) $\boldsymbol{i}$-1.
3) $-{}^3/_2\pi\boldsymbol{i}$.
4) 1.



Method 1.

Parameterize:

$z(t) = e^{-it}$ $t \in [\frac{\pi}{2}, 2\pi]$

$$\int_{\pi/2}^{2\pi} e^{2it}\, d e^{-it} = -i \int_{\frac{\pi}{2}}^{2\pi} e^{it}\, dt =$$

$$-i \frac{e^{it}}{i} \Big|_{\frac{\pi}{2}}^{2\pi} = i-1$$

Method 2

$\bar{z} z = 1$

$$\bar{z}^2 = \frac{1}{z^2} = \left(-\frac{1}{z}\right)'$$

$$\left(-\frac{1}{z}\right)\Big|_{-i}^{1} = i-1$$

Warm up question. Lecture 10.

Thursday, February 11, 2021 1:00 PM

What is the Taylor series of the function $f(z) = ze^z$ at the point $z = 1$?

1) $\sum_{n=0}^{\infty} \frac{1}{n!}(z-1)^n$.

2) $\sum_{n=0}^{\infty} \frac{(1+n)e}{n!}(z-1)^n$.

3) $\sum_{n=0}^{\infty} \frac{e}{(n-1)!}(z-1)^n$.

4) I don't know.

$w = z - 1$

$(w+1)e^{w+1} = e(w+1)e^w =$

$e(w+1)\sum_{n=0}^{\infty} \frac{w^n}{n!} = \sum_{n=0}^{\infty} \frac{w^{n+1}}{n!} + \sum_{n=0}^{\infty} \frac{w^n}{n!}$

$\frac{kw^k}{(k-1)!}$, $k = n+1$

$e\sum_{n=0}^{\infty} \frac{(1+n)w^n}{n!}$

$f^{(n)}(z)\Big|_{z=1}$